



CLOSED BOOK - NO NOTES, NO TEXTS, NO CALCULATORS

Values

[2] 1. (a) How do you find the area of a parallelogram determined by \vec{a} and \vec{b} ?

[2] (b) How do you find the volume of the parallelepiped determined by \vec{a} and \vec{b} and \vec{c} ?

[2] (c) How do you find the angle between two intersecting planes?

[6] (d) If \vec{u} and \vec{v} are differentiable vector functions, c is a scalar, and f is scalar function, write the rules for differentiating
 (i) $\vec{u}(t) + \vec{v}(t)$ (ii) $c\vec{u}(t)$ (iii) $f(t) \vec{u}(t)$
 (iv) $\vec{u}(t) \cdot \vec{v}(t)$ (v) $\vec{u}(t) \times \vec{v}(t)$ (vi) $\vec{u}(f(t))$

[4] (e) (i) What is a smooth curve?
 (ii) How do you find a tangent vector at a point of a smooth curve?
 (iii) How do you find the tangent line?
 (iv) How do you find a unit tangent vector?

[3] (f) (i) Define the linearization of f at (a, b, c) .
 (ii) What is the linear approximation?
 (iii) What is the geometric interpretation of the linear approximation?

[4] (g) Explain how the Lagrange multiplier method works in finding the extreme values of $f(x, y, z)$ subject to the constraint $g(x, y, z) = k$.

[2] (h) Write an expression for the area of a surface with equation $x = f(y, z)$, $(y, z) \in D$.

2. The temperature at a point (x, y, z) is given by

$$T(x, y, z) = 200e^{-x^2-3y^2-9z^2}$$

where T is measured in $^{\circ}\text{C}$ and x, y, z in meters.

[6] (a) Find the rate of change of temperature at the point $P(2, -1, 2)$ in the direction toward the point $(3, -3, 3)$.

[2] (b) In which direction does the temperature increase fastest at P ?

[2] (c) Find the maximum rate of temperature increase at P .

[4] 3. (a) Find an equation for the tangent plane to the ellipsoid $2x^2 + 3y^2 + z^2 = 12$ at the point $(\sqrt{2}, \sqrt{2}, \sqrt{2})$.

[6] (b) Find the points, if any, on the ellipsoid $2x^2 + 3y^2 + z^2 = 12$ where the tangent plane is parallel to the plane $4x - 3y + z = 1$.

[10] 4. Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane $2x + 3y + z = 6$.

[8] 5. Evaluate $\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4} dx dy$ by reversing the order of integration.

[6] 6. (a) Sketch the graphs of $r = 3 \cos \theta$ and $r = 1 + \cos \theta$ and find the points where the graphs intersect.
 (b) Find the area of the region inside $r = 3 \cos \theta$ and outside $r = 1 + \cos \theta$.

[7] 7. Find the work done by the force field

$$F(x, y) = \left(\frac{1}{x + y^2} \right) i + ye^x j$$

in moving an object along the curve with vector equation $r(t) = 4t^2 i + tj$, $1 \leq t \leq 3$.

[8] 8. Find the area of the surface $z = xy$ which lies in the first octant and within the cylinder $x^2 + y^2 = 1$.

[8] 9. Let S denote the solid bounded by the cylinder $y^2 + z^2 = 9$ and the planes $y = 3x$, $y = 0$, $z = 0$ in the first octant. Sketch a picture of S and find its volume.